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Transient conjugated heat transfer in fully developed laminar pipe flows

T. F. LIN and J. C. KUO
Department of Mechanical Engineering, National Chiao Tung University, Hsinchu, Taiwan, Republic of China

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Abstract—This study aims to numerically investigate the effects of conduction heat transfer in the pipe wall on the unsteady forced convective heat transfer in the flow in a long circular pipe resulting from a step change in uniform wall heat flux over a finite length of the pipe. Substantial influences are observed for the variations of Peclet number, radius ratio, conductivity ratio, and diffusivity ratio on the transient heat transfer characteristics. In particular, the wall-to-fluid heat capacity ratio is found to have a decisive impact on the unsteady heat transfer in the flow.

INTRODUCTION

THE GROWING research on unsteady convective heat transfer is mainly stimulated by the increasing need to procure the precise thermal control of various heat exchange devices encountered in chemical processes, nuclear energy systems, and aerospace equipment. Besides, the demands for the detailed understanding of the transient heat transfer characteristics in energy-related systems during the period of start-up, shut-down, or any off-normal surges in a presumed steady normal operation, possibly resulting from the changes in loading conditions, have also significantly increased. In the present study, particular consideration is given to the study of unsteady forced convective heat transfer in laminar flows through long circular pipes with the influences of both heat conduction and heat capacity in the pipe wall taken into account.

In the early attempts to treat the problem, because of the lack of efficient computational tools, approximate methods were employed to deduce the gross features of the unsteady heat transfer [1–7]. Recently, with the availability of large computing systems and efficient numerical schemes, pure numerical solutions have been obtained for transient heat transfer in the thermal entrance regions in laminar duct flows under different thermal conditions [8–10].

In the studies [8–10] just mentioned the transient energy equation for the fluid alone was solved with the thermal boundary condition at the fluid–wall interface prescribed as it is at the outer surface of the pipe wall. The results thus produced are only good for heat transfer in flows bounded by extremely thin walls. In practical situations the pipe wall is normally finite in thickness, and thereby the thermal resistance associated with the conduction heat transfer in the solid wall and the process of thermal energy storing in the wall during the transient state must be included in the analysis. Recognizing this problem, Sucec and Sawant [11–13] improved the analysis and showed that the duct wall heat capacity can have profound influences on the unsteady thermal characteristics. But still the heat conduction in the wall remains untreated, and hence its effect is not known.

In the present study, the system (Fig. 1) to be examined is an infinitely long circular pipe ($-\infty < x < \infty$) with inside radius R_i and outside radius R_o. Initially, the system comprising the flowing fluid and the confining pipe wall is at a constant and uniform temperature T_e. The flow enters the pipe with a uniform velocity u_e and a uniform temperature T_e in the far upstream region ($x \to -\infty$). At time $t = 0$, a uniform heat flux q''_{wo} is suddenly applied at the outer surface of the pipe over a finite length $0 \leqslant x \leqslant l$ and maintained at this level thereafter, while the upstream ($-\infty < x < 0$) and downstream ($l < x < \infty$) regions are well insulated. Attention is focused on the transient thermal interactions between the conduction heat transfer in the pipe wall and the convection heat transfer in the fluid through their common interface. The axial heat conduction, radial heat conduction, and heat capacity effects in both the fluid and the pipe wall on the transient conjugate heat transfer in laminar pipe flows are all rigorously taken into consideration.

In the material that follows the mathematical formulation for the problem is first outlined, the numerical techniques employed to solve the governing differential equations are described next, and finally the results for the unsteady heat transfer characteristics in the system are presented over wide ranges of the controlling non-dimensional groups.

ANALYSIS

Since the fluid is assumed to enter the pipe in the far upstream region, the flow can be regarded as hydrodynamically fully developed in the region where heat transfer is significantly present. By considering

NOMENCLATURE

A wall-to-fluid thermal diffusivity ratio, equations (6)
C_p specific heat
K wall-to-fluid thermal conductivity ratio, equations (6)
k thermal conductivity
L dimensionless heated length, equations (6)
l heated length
Nu Nusselt number, equation (10)
Pe Peclet number, equations (6)
Q_{wi} dimensionless interfacial heat flux, equation (9)
q''_{wi} interfacial convective heat flux, equation (11)
q''_{wo} input heat flux to the outside surface of the pipe
R_i inside radius
R_o outside radius
r radial coordinate
T temperature
t time
u fluid axial velocity
x axial coordinate.

Greek symbols
α thermal diffusivity
β ratio of outside and inside radii, equations (6)
η dimensionless radial coordinate, equations (6)
θ dimensionless temperature, equations (6)
ξ dimensionless axial coordinate, equations (6)
ρ density
τ dimensionless time, equations (6).

Subscripts
b bulk quantity
e initial value at the entrance of the pipe
f fluid
w pipe wall
wi fluid–wall interface.

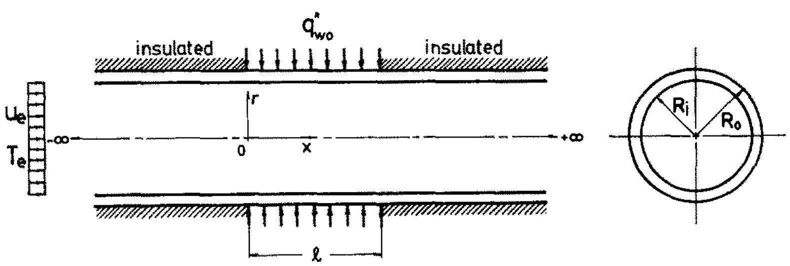

FIG. 1. Schematic diagram of the physical system.

the thermophysical properties of the fluid and wall to be temperature independent, the energy transport processes in the system are governed by the following non-dimensional equations as:

energy equation for the fluid

$$\frac{\partial \theta_f}{\partial \tau} + Pe(1-\eta^2)\frac{\partial \theta_f}{\partial \xi} = \frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \theta_f}{\partial \eta}\right) + \frac{\partial^2 \theta_f}{\partial \xi^2}; \quad (1)$$

energy equation for the wall

$$\frac{\partial \theta_w}{\partial \tau} = A\left[\frac{1}{\eta}\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \theta_w}{\partial \eta}\right) + \frac{\partial^2 \theta_w}{\partial \xi^2}\right]. \quad (2)$$

The initial conditions are

$$\tau = 0, \quad \theta_f = \theta_w = 0, \quad -\infty < \xi < \infty. \quad (3)$$

The basic equations are subjected to the following boundary conditions:

$$\eta = 0, \quad \frac{\partial \theta_f}{\partial \eta} = 0, \quad -\infty < \xi < \infty$$

$$\eta = \beta, \quad \frac{\partial \theta_w}{\partial \eta} = 1/K, \quad 0 \leqslant \xi \leqslant L$$

$$0, \quad \text{otherwise}$$

$$\xi \to -\infty, \quad \theta_f = \theta_w = 0$$

$$\xi \to \infty, \quad \frac{\partial \theta_f}{\partial \xi} = \frac{\partial \theta_w}{\partial \xi} = 0. \quad (4)$$

The continuity of temperature and heat flux at the wall–fluid interface is described by

$$\eta = 1, \quad \theta_f = \theta_w,$$

$$\frac{\partial \theta_f}{\partial \eta} = K\frac{\partial \theta_w}{\partial \eta}, \quad -\infty < \xi < \infty. \quad (5)$$

Here the dimensionless quantities are defined as

$$\xi = x/R_i$$

$$\eta = r/R_i$$

$$\tau = t/(R_i^2/\alpha_f)$$

$$L = l/R_i$$
$$\beta = R_o/R_i$$
$$\theta = (T-T_e)/(q''_{wo}R_i/k_f)$$
$$Pe = 2R_i u_e/\alpha_f$$
$$K = k_w/k_f$$
$$A = \alpha_w/\alpha_f \quad (6)$$

where the various symbols used in the above equations are defined in the Nomenclature.

It is clear in the above formulation that five governing non-dimensional groups appear for the problem, namely, the Peclet number of the flow Pe, the wall-to-fluid thermal diffusivity ratio A, the wall-to-fluid thermal conductivity ratio K, the ratio of the outside and inside radii β, and the dimensionless heating length L. Their influences on the transient thermal interactions between the fluid and the wall will be examined in detail.

The interfacial temperature θ_{wi}, bulk fluid temperature θ_b, dimensionless interfacial heat flux Q_{wi} and local Nusselt number Nu are of major interest for a designer of thermal systems. They can be evaluated as follows:

$$\theta_{wi} = \theta_f(\xi, 1) = \theta_w(\xi, 1) \quad (7)$$

$$\theta_b = \int_0^1 (\eta-\eta^3)\theta_f \,d\eta \bigg/ \int_0^1 (\eta-\eta^3)\,d\eta \quad (8)$$

$$Q_{wi} = \frac{R_i}{R_o}\frac{q''_{wi}}{q''_{wo}} = \frac{K}{\beta}\frac{\partial\theta_w}{\partial\eta}\bigg|_{\eta=1} \quad (9)$$

$$Nu = 2\beta Q_{wi}/(\theta_{wi}-\theta_b) \quad (10)$$

where

$$q''_{wi} = k_w \frac{\partial T_w}{\partial r}\bigg|_{r=R_i}. \quad (11)$$

SOLUTION METHODOLOGY

On account of the interactions between the convection heat transfer in the flow and the conduction heat transfer in the pipe wall across the fluid–wall interface, the solutions of this problem defined by the foregoing equations can be found better by numerical finite-difference procedures. Because equations (1) and (2) are elliptic in space and parabolic in time, the solution can be marched in time and swept in space from upstream to downstream of the heated region with iterations. A fully-implicit numerical scheme in which the terms representing the fluid energy transport by conduction and convection are approximated by the exponential scheme [14, 15], the wall diffusion terms by the conventional central difference, and the unsteady terms by the backward difference, is employed to transform the governing equations into finite-difference equations.

It is noted that the convection terms has an inseparable connection with the diffusion term for the fluid energy equation, and thus two terms need to be handled as one unit by using the exponential scheme. To be more specific, equation (1) can be written as

$$\left(\frac{\theta_{i,j}^{k+1,n}-\theta_{i,j}^{k,n}}{\Delta\tau_k}\right)\Delta\tilde{\xi}_i\Delta\tilde{\eta}_j + J_{i+1/2,j} - J_{i-1/2,j} + J_{i,j+1/2} - J_{i,j-1/2} = 0 \quad (12)$$

where

$$J_{i+1/2,j} = Pe(1-\eta_j^2)\left[\theta_{i,j}^{k+1,n} + \frac{\theta_{i,j}^{k+1,n}-\theta_{i+1,j}^{k+1,n}}{\exp(P_i)-1}\right]\Delta\tilde{\eta}_j$$

$$J_{i-1/2,j} = Pe(1-\eta_j^2)\left[\theta_{i-1,j}^{k+1,n} + \frac{\theta_{i-1,j}^{k+1,n}-\theta_{i,j}^{k+1,n}}{\exp(P_{i-1})-1}\right]\Delta\tilde{\eta}_j$$

$$J_{i,j+1/2} - J_{i,j-1/2} = \left[-\left(\frac{\partial^2\theta}{\partial\eta^2}\right)_{i,j}^{k+1,n} - \left(\frac{1}{\eta}\frac{\partial\theta}{\partial\eta}\right)_{i,j}^{k+1,n}\right]\Delta\tilde{\xi}_i\Delta\tilde{\eta}_j. \quad (13)$$

In the above equation J is the total heat flux, $\theta_{i,j}^{k+1,n}$ the dimensionless temperature at nodal point (i,j) at the $k+1$th time step at iteration n, and i, j, k, and n are the indices in the axial direction, radial direction, time step, and iteration, respectively

$$P_i = Pe(1-\eta_j^2)\Delta\tilde{\xi}_i, \quad \Delta\tilde{\eta}_j = \tfrac{1}{2}(\eta_{j+1}-\eta_{j-1}),$$
$$\Delta\tilde{\xi}_i = \tfrac{1}{2}(\xi_{i+1}-\xi_{i-1}). \quad (14)$$

Each system of finite-difference equations forms a tridiagonal matrix which can be efficiently solved by the Thomas algorithm [15]. For a given condition, a brief outline of the solution procedures is described as follows.

(1) Solve governing equations for θ_f and θ_w by the line-by-line method [15] from far upstream to far downstream of the heating section where the downstream boundary condition prescribed in equation (4) is met for a given time instant.

(2) Check whether the sum of the relative error of θ_f and θ_w for each iteration is small enough or not. If yes, the solution for the current time step converges. Proceed to the next time instant. If not, repeat the above procedure for the current time step until convergent solutions are reached.

(3) Procedures (1) and (2) are repeatedly applied to each time step from the start of the transient to the instant at which the steady state is achieved.

To obtain desired accuracy, non-uniform grids were employed to account for the uneven variations of θ_f and θ_w in space and in time with non-uniform time steps. In the radial direction the grid point density is highest near the interface, whereas in the axial direction the highest concentration of the grid point is around the beginning and exit end of the heating section.

To verify the adequacy of the numerical scheme just described for the problem considered, the results for the unsteady axial variations of interfacial temperature are obtained by the present numerical scheme in the limiting case in which the Peclet number, wall-to-fluid conductivity ratio, and heating length are very large, and wall thickness is very small and by the scheme employed by Chen *et al.* [9]. Excellent agreement is observed. This comparison lends support for the use of the scheme proposed to the analysis of the present problem.

DISCUSSION OF RESULTS

The preceding analysis indicates that the transient heat transfer characteristics in the flow and pipe wall depend on the Peclet number Pe, the wall-to-fluid thermal conductivity ratio K, the wall-to-fluid thermal diffusivity ratio A, the ratio of outside and inside radii β, and the dimensionless length of the heated section L. While computations can be conducted for any combination of these parameters, the objective here is to present a sample of results that could illustrate the phenomena of the transient conjugate heat transfer which takes into account the heat capacity effects, axial and radial conduction in the flow and pipe wall. In the following, attention is primarily given to a system in which liquid metal flows in a metal pipe.

As was explained by Faghri and Sparrow [16], the distribution of Nusselt number is not very informative in the study of conjugate heat transfer. Instead, the interfacial heat flux distribution contains more information. The unsteady axial distributions of the non-dimensional interfacial heat flux Q_{wi} are shown in Fig. 2 for $Pe = 20$, $K = 1$, $\beta = 1.5$, $A = 4$, and $L = 50$ at various instants of time. A number of interesting features are unveiled in this figure. In the immediate upstream region of the heated section where no energy is directly supplied to the pipe from the outer surface,

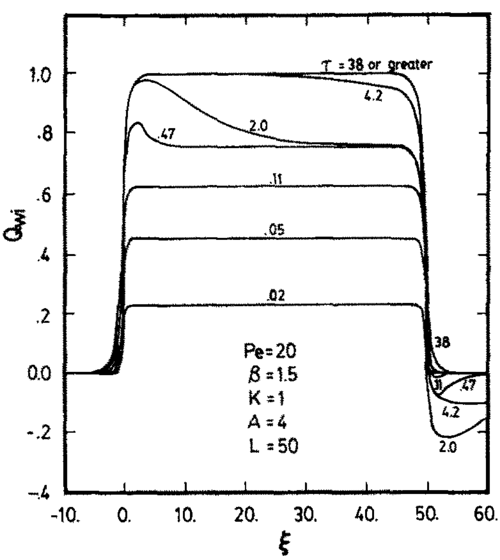

FIG. 2. Transient axial distributions of interfacial heat flux.

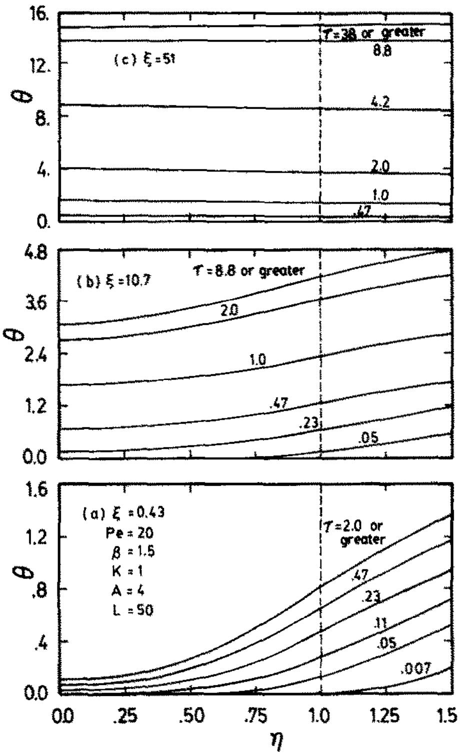

FIG. 3. Time evolution of temperature profiles at various axial locations.

some heat transfer from the wall to the fluid is noted. The occurrence of this unusual heat transfer is due to the heat penetration by the upstream axial conduction heat transfer in the wall from the direct heating region to the upstream unheated region [16]. As time goes, the thermal energy diffuses further upstream, and its magnitude becomes larger. The curves of Q_{wi} drop off with increasing upstream distance as the thermal energy stored in the wall is drained away by the flowing fluid across the interface. In the region of direct heating, Q_{wi} increases with time and gradually reaches the steady state condition. As a result it takes a finite period of time for the energy applied on the outer surface of the pipe to be transferred to the inner surface. This is a direct consequence of the finite heat capacity of the wall material and is commonly known as the thermal lag of the system. If the wall was neglected in the analysis, Q_{wi} would be unity in the direct heating region and zero in the other regions. No thermal lag can occur.

As τ is small, Q_{wi} grows very quickly and is rather uniform in the axial direction except near the ends of the heated section. The uniformity of Q_{wi} is caused by the dominant radial heat conduction over the axial forced convection during the initial transient ($\tau < 0.11$). Hence the temperature variations are mainly in the solid region, as evident in the temperature profiles shown in Fig. 3 where the temperature profiles at three different axial locations, $\xi = 0.43$, 10.7, and 51, are plotted at various instants of time. The period of the initial transient can be approximately estimated from the smaller of the two criteria—one based on the axial convection as $\tau < \xi < Pe$ and another on the radial conduction as $\tau < (\beta-1)/2\sqrt{A}$. After $\tau = 0.11$, the forced convection exhibits an increasing influence on heat transfer processes, which results in the presence of a peak of Q_{wi} near $\xi = 0$. With further increase in time, the peak becomes more pronounced. In the meantime the fluid carrying the convective energy downstream gradually levels Q_{wi} off until the steady state is reached, at this state Q_{wi} is unity in the major portion of the direct heating region.

In the neighborhood of the exit end of the heated section, Q_{wi} shows a dramatic change in the flow direction at different moments of time. At the very beginning of the heating, only a small amount of energy is transported away by the axial conduction in the pipe wall to the unheated portion of the pipe so that Q_{wi} is infinitesimal in quantity. With time elapsing, the energy transported by axial conduction in the pipe wall downstream gradually increases, whereas the heat transported by the forced convection in the flow has not arrived at this region. Consequently, heat penetration into the downstream region through the wall and transfer to the flow across the interface can be observed. As time increases further, it is surprising to find that Q_{wi} is negative, i.e. heat transfer is from the fluid to the wall. The reversal in heat transfer direction may be brought about by the possibility that the propagation speed of heat transfer by axial conduction in the solid wall from the adjacent heated section can be slower than the energy transport in the flow by forced convection from the flowing fluid in the heated section at a certain time instant, and hence the wall temperature is below the fluid temperature, as evident from the close inspection of the temporal evolution of the temperature profiles given in Fig. 3(c) for the curve for $\tau = 2.0$. Accordingly heat is transferred from the flow to the wall. At a certain later instant of time, the temperature penetration in the postheated region by the heat conduction in the wall can be large enough so that the wall temperature becomes higher than the fluid temperature. The heat transfer is then from the wall to the fluid.

It is also noteworthy in Fig. 2 that, near the beginning of the heated section, Q_{wi} gets to the steady state value quickly, which is more clearly shown in Fig. 3. Steady state is already prevalent when $\tau = 2.0$ at $\xi = 0.43$, while at $\xi = 10.7$ and 51, the steady states are reached at $\tau = 8.8$ and 38, respectively. By inspecting the governing equations given in equations (1) and (2), it can be stated that the condition of the steady heat transfer is attained when, considering a differential control volume in the flow, the convection energy transport in the axial direction is balanced by the diffusional energy transport in both the axial and radial directions in the flow. In the meantime, the conduction heat transfer is balanced in both radial and axial directions in the pipe wall. Consequently, no net energy is stored in the fluid and pipe wall.

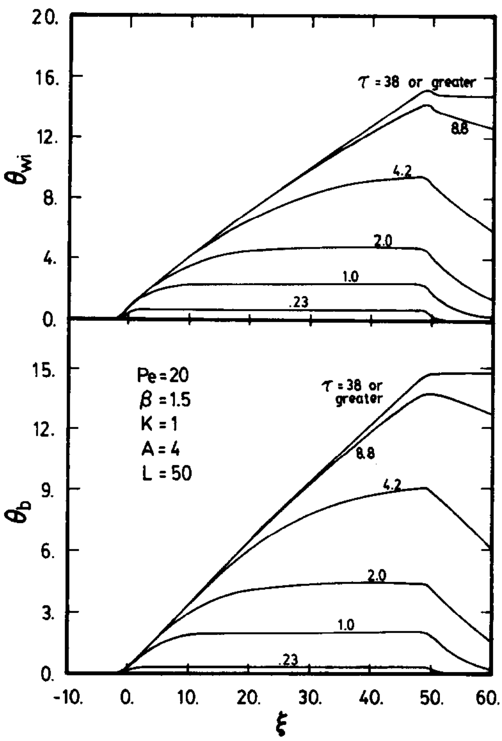

FIG. 4. Unsteady axial variations of bulk fluid and interfacial temperature.

Aside from the dimensionless interfacial heat flux which is important in the aspect of thermal interactions between the fluid and the pipe wall, the characteristics of the transient axial distributions of the bulk fluid and interfacial temperatures are more descriptive from the viewpoint of understanding the energy transport processes in the system. Hence, to improve our understanding, Fig. 4 presents the results for the distributions of the bulk fluid and interfacial temperatures at various instants of time. The aforementioned characteristics, namely, the domination of the heat conduction effect at small τ, the early appearance of the steady state condition at the upstream station of the direct heating region, the phenomena of heat penetration from the heated region to the unheated region, and the change of heat transfer direction immediately downstream of the exit end of the heating section are all clearly illustrated in this figure. In the early period, the heat conduction plays an important role so that the curves for θ_{wi} and θ_b become horizontal in the region of direct heating. But the quantity of heat transported by axial conduction into the unheated region is very small so that the values of θ_{wi} and θ_b are infinitesimal in the unheated portion near the entering and exit ends of the direct heating region. As time goes on, the influences of both the forced convection in the flow and the conduction in the pipe wall increase so that θ_{wi} and θ_b increase with ξ at the initial portion of the region. This portion gets larger with time. Finally, a nearly linear increase of the temperatures results when

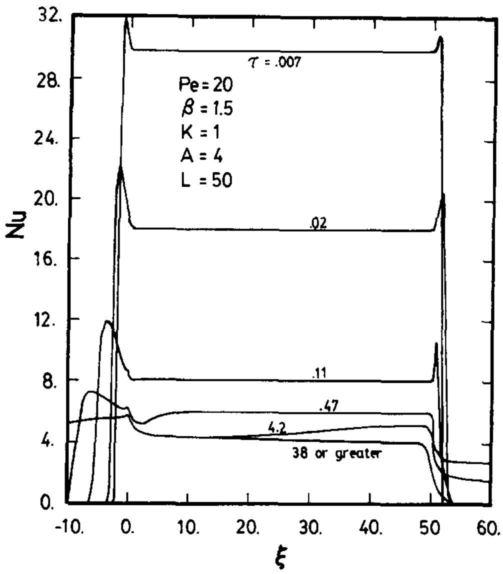

FIG. 5. Unsteady axial distributions of local Nusselt number.

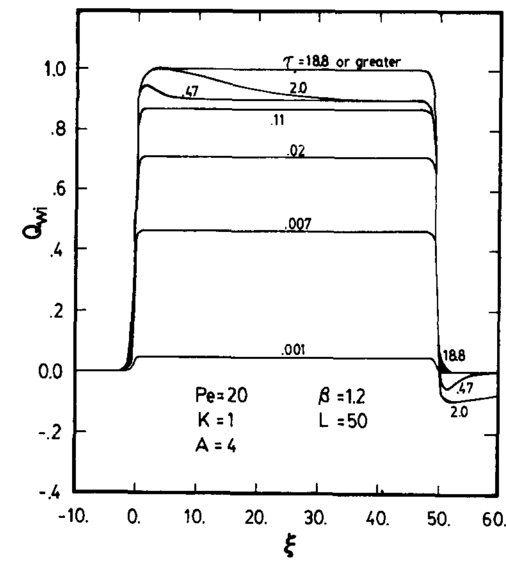

FIG. 6. Effects of the pipe wall thickness on the transient axial distributions of interfacial heat flux.

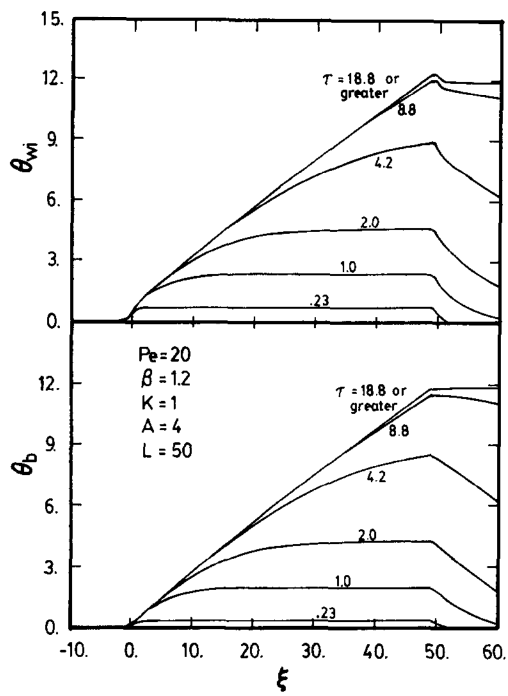

FIG. 7. Effects of the pipe wall thickness on the unsteady axial variations of bulk fluid and interfacial temperatures.

the steady state condition is approached. In the neighborhood of the exit end of the heating section, decreases in θ_b and θ_{wi} are noted. This decrease in temperatures is due to the energy drained away in the flow direction by the heat conduction in the pipe wall.

As was mentioned earlier, the local Nusselt number in the flow is not a convenient computational parameter for the conjugate heat transfer problem because it includes three unknowns—Q_{wi}, θ_{wi} and θ_b. Besides its magnitude does not represent the actual heat transfer rate. In spite of these defects, the distributions of *Nu* are shown here for those who are interested in the thermal system design. The unsteady axial distributions of *Nu*, shown in Fig. 5, are quite different from those predicted with the wall effects neglected [8–10]. By and large *Nu* decreases with time in the direct heating region. This is simply because at short time the very small difference between θ_b and θ_{wi} results in a large value of *Nu*, according to the definition of *Nu*, equation (10).

To investigate the effects of the pipe wall thickness, the unsteady axial distributions of Q_{wi} are displayed in Fig. 6 for $\beta = 1.2$. These curves resemble those in Fig. 2. But there are several prominent differences between them. The time required for the heat transfer in the system to arrive at the steady state condition is less for the case with $\beta = 1.2$. This point can be made clearer by comparing Figs. 4 and 7 where the axial distributions of θ_{wi} and θ_b for $\beta = 1.5$ and 1.2 are plotted at various τ. Besides, it is also found that the value of Q_{wi} at $\tau = 0.11$, when the heat conduction is still dominant, is larger for the case with $\beta = 1.2$. The above outcomes are apparently due to the fact that the total thermal resistance and heat capacity of the wall are small for a thinner wall so that the heat supplied from the outside surface of the pipe is easily transported to the wall and the fluid. Consequently, the presence of the pipe wall has a significant influence on the characteristics of heating the flowing fluid during the unsteady state, and thus the wall effects cannot be disregarded for transient conjugate heat transfer problems.

Next, the influences of the Peclet number of the flow on the characteristics of the transient heat transfer are examined. Illustrated in Figs. 8 and 9 are the time evolutions of the axial distributions of Q_{wi}, θ_{wi} and θ_b, respectively, for $Pe = 100$. In appearance these curves are similar to those presented in Figs. 2 and 6 for Q_{wi}

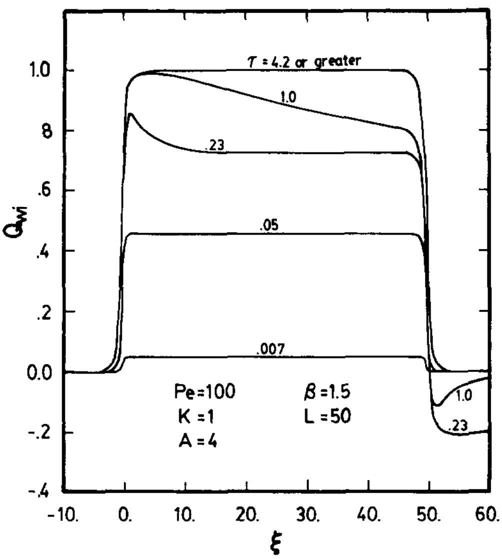

FIG. 8. Effects of the Peclet number on the transient axial distributions of interfacial heat flux.

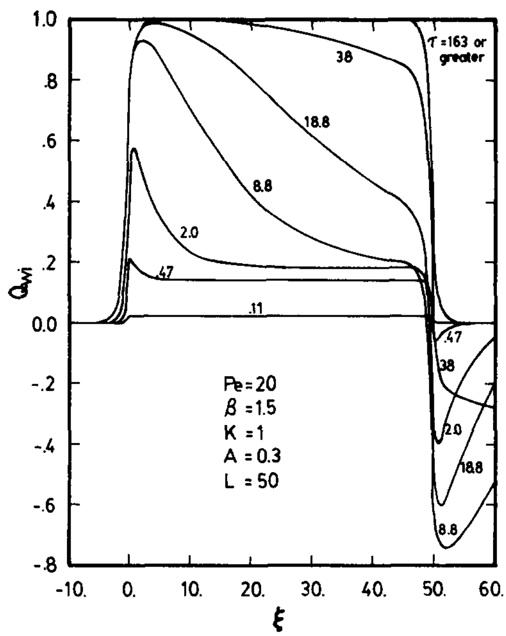

FIG. 10. Effects of the thermal diffusivity ratio on the transient axial distributions of interfacial heat flux.

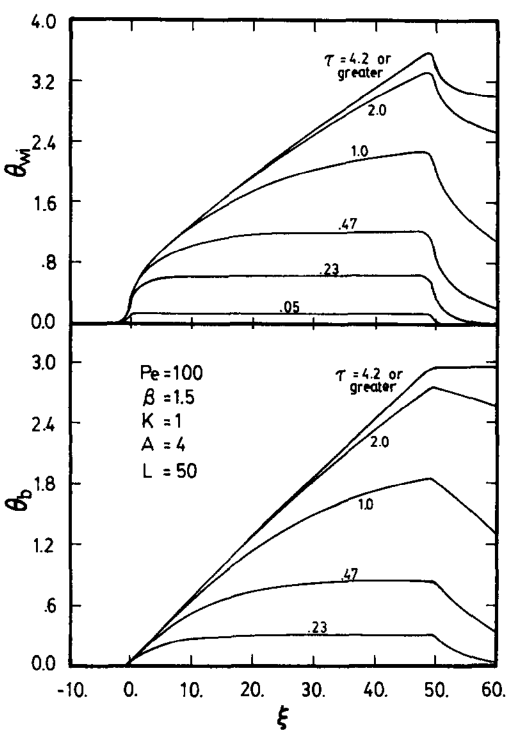

FIG. 9. Effects of the Peclet number on the unsteady axial variations of bulk fluid and interfacial temperatures.

and those in Figs. 4 and 7 for θ_{wi} and θ_b. Careful scrutiny of those plots reveals the effect of the pipe wall thickness, the time required for heat transfer in this system to attain the steady state condition is shorter for the case with $Pe = 100$. The result may be caused by the fact that as Pe gets larger, the thermal energy stored in the pipe wall is more quickly carried away downstream by the fluid at a higher velocity. Examining the scales of the ordinates in the plots (Figs. 4 and 9) indicates that θ_{wi} and θ_b are smaller for the case with $Pe = 100$, as compared to those with $Pe = 20$. In addition, the negative Q_{wi} in the neighborhood of the exit end of the directly heated section can exist in a larger portion of the flow for larger Pe, and the heat transfer reversal takes place earlier.

In Fig. 10, the effect of the wall-to-fluid thermal diffusivity ratio on the distributions of Q_{wi} are presented for $A = 0.3$ at various instants of time. These curves are very different from those in Fig. 2. Comparing the results in Figs. 2 and 10 clearly shows that the time from the start of the transient to the steady state is much longer for the case with $A = 0.3$. This difference is believed to result from the fact that for a small A ($= \alpha_w/\alpha_f$), or equivalently a small α_w or a large wall heat capacity in a relative sense, the heat transmission in the wall by the conduction heat transfer during the unsteady temperature change is then slower [12]. This in turn causes a larger thermal lag in the system, as is apparent by comparing Figs. 4 and 11. Special attention is focused on the curves of θ_{wi} and θ_b for smaller τ shown in Figs. 4 and 11. The values of θ_{wi} and θ_b at $\tau = 1.0$ for the case with $A = 0.3$ are smaller than those for the case with $A = 4$, since at this time instant the conduction dominates the heat transfer process for $A = 0.3$ while the convection already shows up in the flow for $A = 4$. Moreover, there is a prominent difference between the results for Q_{wi} in Figs. 2 and 10—the magnitude of the negative Q_{wi} in the unheated portion around the exit end of the heating section for the case with $A = 0.3$ is larger than that for the case with $A = 4$. The larger heat transfer reversal is due to the fact that for smaller A the heat capacity of the wall is larger, and thus the temperature

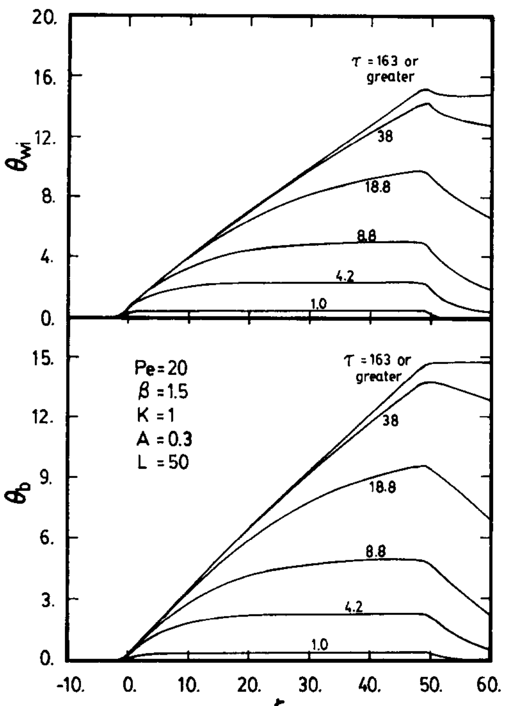

FIG. 11. Effects of the thermal diffusivity ratio on the unsteady axial variations of bulk fluid and interfacial temperatures.

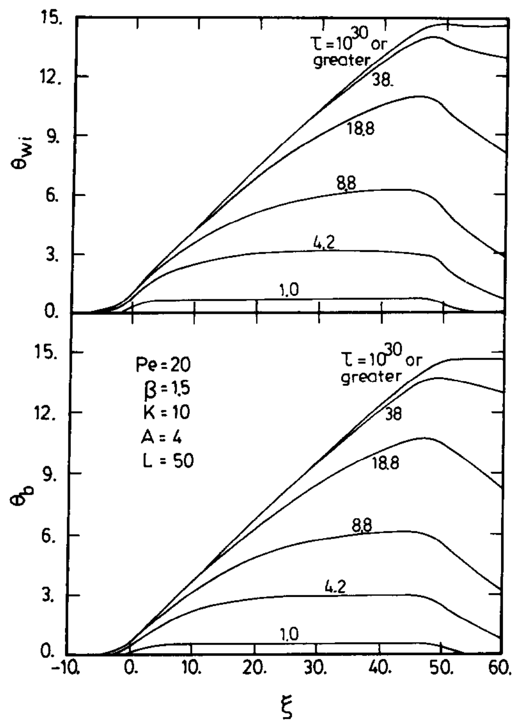

FIG. 13. Effects of the thermal conductivity ratio on the unsteady axial variations of bulk fluid and interfacial temperatures.

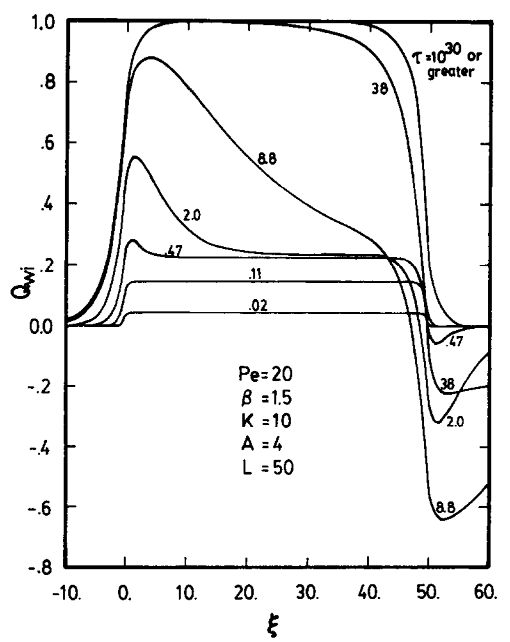

FIG. 12. Effects of the thermal conductivity ratio on the transient axial distributions of interfacial heat flux.

penetration by the axial conduction in the unheated solid wall from the adjacent heated section is much slower than the energy transport by the forced convection from the flowing fluid in the heating section.

The influences of the wall-to-fluid thermal conductivity ratio on the characteristics of the transient heat transfer are also interesting. Plotted in Figs. 12 and 13 are the axial distributions of Q_{wi}, θ_b and θ_w at various instants of time for $K = 10$. These curves are similar to those in Figs. 2, 6, 8 and 10. But there are two noticeable differences between them. First, for $K = 10$ the extent of the temperature penetration through wall conduction upstream and downstream of the direct heating region is more substantial. Secondly, the time needed for the heat transfer transient in this system to die away is a lot longer for the case with $K = 10$. For instance, it can be directly read from the plots that at $\xi = 20$ the steady state is not reached as $\tau = 38$ for $K = 10$, while for $K = 1$ at the same axial location the steady state is already achieved when $\tau = 4.2$. This result can be readily understood by recognizing that for a larger K $(= A\rho_w c_{pw}/\rho_f c_{pf})$ with A fixed, the heat capacity for the wall $\rho_w c_{pw}$ is larger by comparing with that for the fluid. Like the effects of lowering A, this increase in K results in not only a longer thermal time lag but also a larger negative Q_{wi} in the unheated portion in the exit end of the heating section.

The results presented above for the effects of A and K on the transient heat transfer characteristics seem to suggest that the ratio K/A $(= \rho_w c_{pw}/\rho_f c_{pf})$ is a major parameter on the thermal lag of the system, which is consistent with the simplified analysis given by Sucec [11].

The last governing parameter on the unsteady thermal characteristics is the heating length. Its effect on the unsteady heat transfer in the system is relatively insignificant except for the case with a short heating length ($L < 10$) for which the thermal interactions between the upstream and downstream regions of the heating section may appear.

CONCLUDING REMARKS

Transient conjugate heat transfer considering heat capacity and conduction heat transfer in both axial and radial directions in the fluid and in the pipe wall as well as in fully-developed laminar pipe flows resulting from a step change in a uniform wall heat flux over a finite length of the pipe has been numerically studied. Results are particularly presented for the heating of liquid metal flowing in a metal pipe over wide ranges of the governing parameters. The influences of five governing non-dimensional groups, i.e. the Peclet number Pe, the ratio of outside and inside radii β, the wall-to-fluid thermal conductivity ratio K, the wall-to-fluid thermal diffusivity ratio A, and the non-dimensional heating length L, have been investigated in great detail.

The results obtained in the present study can be briefly summarized as follows.

(1) The unsteady axial variations of non-dimensional interfacial convective heat flux considerably deviate from the corresponding steady values especially in the initial transients.

(2) The existence of peaks of Q_{wi} in the direct heating region around $\xi = 0$ and the appearance of negative Q_{wi} in the unheated region immediately following $\xi = L$ during the unsteady state are observed.

(3) The wall plays a significant role in a transient conjugate heat transfer problem.

(4) The Peclet number has significant influences on the unsteady heat transfer. It takes less time for the transient to die out for the flow with larger Peclet number.

(5) The heat capacity of the wall has a decisive effect with regard to the speed of propagation of thermal energy from the outer surface of the pipe to the wall–fluid interface.

(6) The time required for the heat transfer to reach the steady state condition is longer for the system with large β, K, and L or with smaller Pe and A.

The numerical scheme employed in the present study is expected to be applicable to any flowing fluid–pipe wall combinations although it is used here to obtain the transient heat transfer results for the liquid metal flow in a metal pipe. It is of interest to find out how important the presence of a finite pipe wall is on the convection heat transfer in other combinations. Also the extension of the present study to the turbulent flows is certainly of great value. Moreover, the inclusion of the buoyancy effects in the study is highly recommended because they always exist in the flows.

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TRANSFERT THERMIQUE CONJUGUE VARIABLE DANS DES ECOULEMENTS LAMINAIRES ETABLIS DANS DES TUBES

Résumé—On étudie numériquement les effets de la conduction thermique dans la paroi du tube sur le transfert convectif forcé par l'écoulement et résultant d'un changement échelon d'un flux de chaleur uniforme à la paroi, sur une longueur finie du tube. On observe des effets substantiels suivant les valeurs du nombre de Peclet, du rapport du rayon, des rapports de conductivité et de diffusivité, sur les caractéristiques du transfert thermique variable. En particulier, le rapport des capacités calorifiques a un effet important sur le transfert thermique variable dans l'écoulement.

INSTATIONÄRE KONJUGIERTE WÄRMEÜBERTRAGUNG IN VOLLENTWICKELTEN LAMINAREN ROHRSTRÖMUNGEN

Zusammenfassung—Das Ziel dieser Arbeit ist die numerische Untersuchung des Einflusses der Wärmeleitung in der Rohrwand auf den instationären Wärmeübergang bei erzwungener Konvektion in einem langen Kreisrohr. Dieser Einfluß resultiert aus einer sprunghaften Änderung des Wärmestroms durch die Wand entlang eines endlichen Rohrabschnitts. Einen wesentlichen Einfluß auf den instationären Wärmeübergang haben die Peclet-Zahl, das Radienverhältnis, das Verhältnis der Wärmeleitfähigkeiten und das Verhältnis der Temperaturleitfähigkeiten. Insbesondere hat das Verhältnis der Wärmekapazitäten von Wand und Fluid einen entscheidenden Einfluß auf den instationären Wärmeübergang in der Strömung.

НЕУСТАНОВИВШИЙСЯ СОПРЯЖЁННЫЙ ТЕПЛОПЕРЕНОС ПРИ ПОЛНОСТЬЮ РАЗВИТОМ ЛАМИНАРНОМ ТЕЧЕНИИ В ТРУБЕ

Аннотация—Численно исследовано влияние кондуктивного переноса тепла в стенке трубы на нестационарный конвективный теплоперенос внутри длинной круглой трубы при ступенчатом изменении однородного теплового потока на участке конечной длины. Обнаружено существенное влияние изменения числа Пекле, отношения радиусов, коэффициентов теплопроводностей и температуропроводностей на характеристики неустановившегося теплообмена. В частности найдено, что отношение теплоёмкостей стенки и жидкости оказывает существенное влияние на нестационарный теплообмен в потоке.